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WAVE TURBULENCE AND SOLITON DYNAMICS

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13. ABSTRACT *(Maximum 200 words)*

Work in three areas is summarized in this report. 1) Two new localized structures were experimentally discovered in vibrating lattices and were described theoretically with a nonlinear Schrodinger equation. One is a domain wall between different types of vibration, the other is a kink in the phase of vibration. The kink has also been discovered in parametrically driven surface waves on water. 2) The direction and spectral energy of interacting nonlinear ocean waves evolve as they approach a sloping beach. A first principles theory was developed for this process and it was shown to agree with ocean experiments. 3) Experiments to determine whether random interacting waves on the ocean move collectively, having average quantities analogous to "pressure" in a gas as well as collective modes of vibration, are described.

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To the Office of Naval Research

WAVE TURBULENCE AND SOLITON DYNAMICS

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Funds from this work order were used to support the labor costs of Dr. Bruce Denardo and Adj. Prof. Andres Larraza, who were essentially co-principle investigators in this work and who wrote most of this report, with some funds also supporting Assist. Prof. Robert Keolian. Our work was in three areas:

1) Localized structures in vibratory media,

2) Nonlinear transformation of directional wave spectra in shallow water, and

3) Wave turbulence,

which are described below.

1. Localized Structures in Vibratory Media

We have studied self-localized standing wave modes in two types of nonlinear media: lattices and surface waves on a liquid. Investigations of lattices have led us to the discovery of domain walls and kinks (Fig. 1). Experimental observations were made in a lattice of coupled pendulums, and numerical simulations in simplified models yielded essentially identical results. A nonlinear Schrödinger (NLS) theory was developed for kinks in the upper cutoff mode (in which each oscillator is out-of-phase with its immediate neighbors), and the theory agrees well with observational data. The domain walls and noncutoff kinks are new localized structures, and may lead to new generic equations at the level of the NLS, Korteweg-de Vries, sine-Gordon, and Toda equations. To our knowledge there are no reported observations or theory of vibratory kinks and domain walls. A manuscript, "Observations of localized structures in nonlinear lattices: Domain walls and kinks," regarding the investigations has appeared in *Physical Review Letters* (1992). This work was done with a graduate student (Lt. Brian Galvin, USN), and in collaboration with Prof. Seth Putterman and two graduate students at UCLA.

Our lattice studies led us to the prediction of fundamentally new modulational kinks in a class of model continua. Although surface waves lie outside this class, we have theoretically shown that these kinks should also exist in this case (Fig. 2). We recently observed such structures in a vertically-oscillated annular channel of liquid. For sufficiently large amplitude, the kink occurs during the transition from one mode to the next as the frequency is slowly increased or decreased. At smaller amplitudes, the transition occurs uniformly. The experimental work is being done with a graduate student (Capt. Charles McClelland, USMC), and the surface wave theoretical work is being done by us. The model continuum work, which will be published separately, is being done in collaboration with UCLA Profs. Seth Putterman and Paul Roberts. Future work includes an investigation of the existence of these kinks in optical fibers, and whether they would offer advantages over the conventional NLS solitons

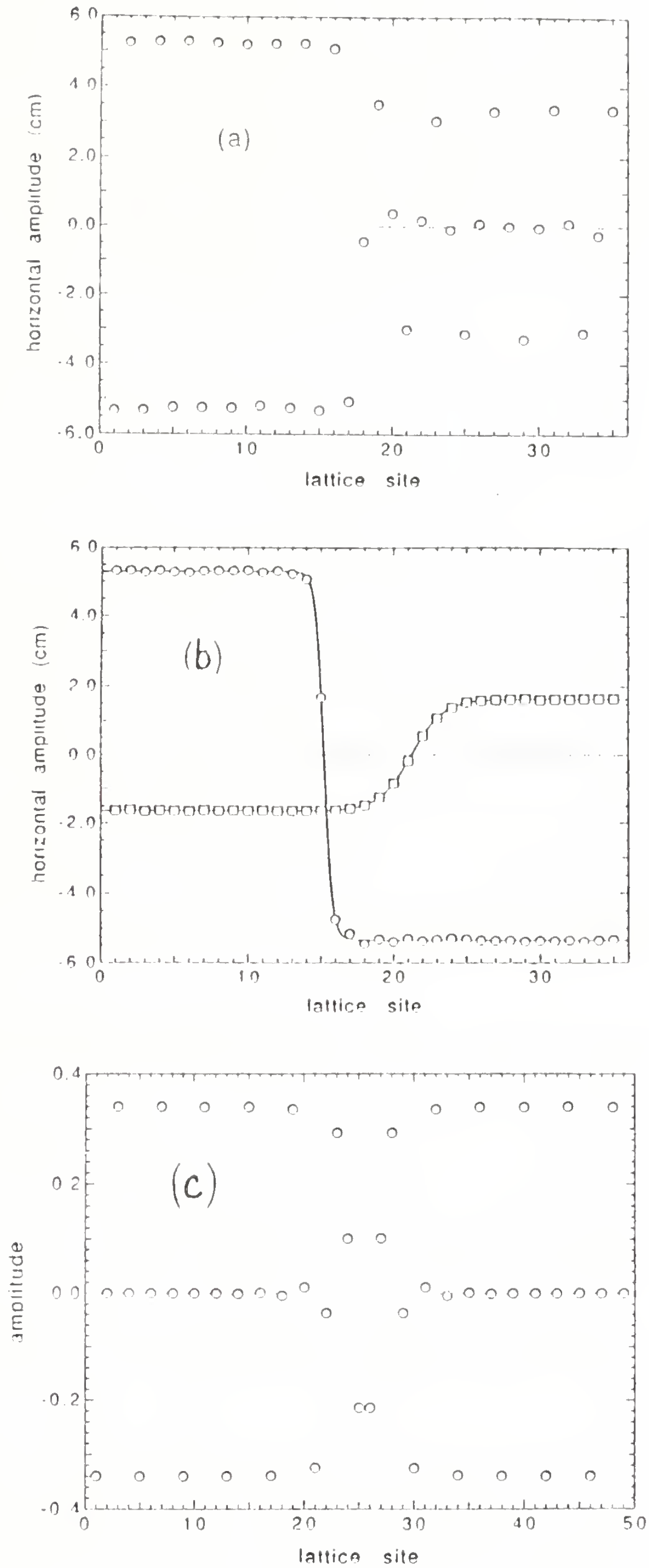

Fig. 1 Localized structures in nonlinear vibratory lattices: (a) domain wall, (b) processed upper cutoff kink data, and (c) kink in a noncutoff mode. The data in (a) and (b) are from an experimental lattice. The data in (c) are from a numerical model. In (b), the kink data have been divided by pure mode data to smooth out the effects of nonuniformity.

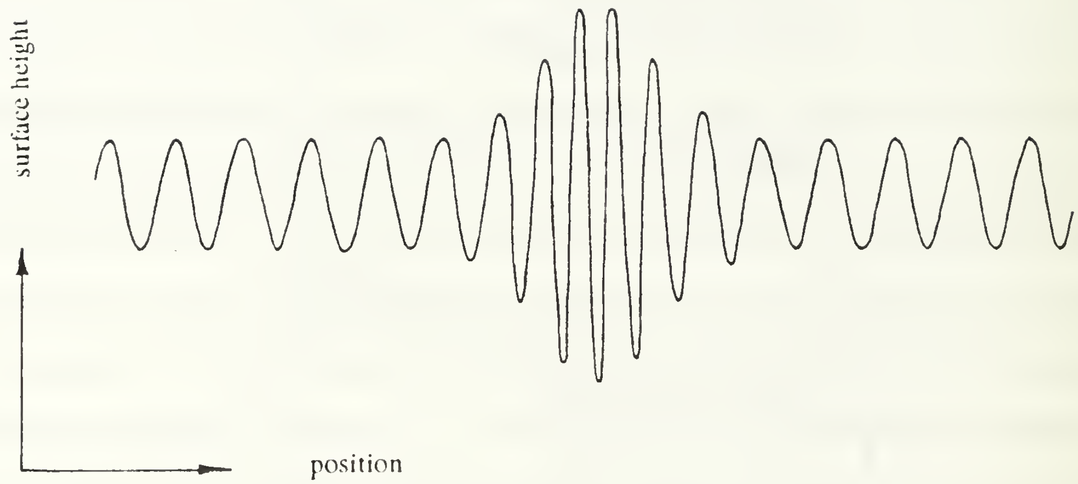

Fig. 2 Theoretical standing surface wave kink. Both the amplitude and wavelength are modulated.

planned for use in fiber optic communications (Mollenauer, 1986).

2. Nonlinear transformation of directional wave spectra in shallow water.

Directional wave spectra give a complete description of the frequency and directional spreading of the ocean wave field. As surface gravity waves propagate shoreward in shoaling waters, linear and nonlinear processes act simultaneously to transform substantially their frequency and directional characteristics.

Changing bottom topography causes refraction and shoaling of the wave field, that result in spatial variations in the amplitudes and directions. Linear refraction theory has been extensively used to estimate the evolution of a shoaling wave field (Longuet-Higgins, 1957; Izumiya and Horikawa, 1987), with reasonable success in predicting the directional characteristics of the waves in shallow water. However, ocean surface gravity waves are essentially nonlinear, and the evolution of ocean wave spectra in shallow water is determined to a considerable extent by the energy flux between spectral wave components, as shown by the observations of Freilich, Guza and Elgar (1990). In the shallow water regime three-wave resonant interactions are possible. These nonlinearities, of lower order than the four wave processes, dominate the flux of energy across the spectrum. The consequences of the transfer of energy associated with resonant wave interactions are not only distortions of the frequency spectrum, but also alterations of the directional spreading of energy. Unlike previous studies, energy transfers across the spectrum by triad interaction were considered within a collision integral formulation.

Starting from a Hamiltonian formalism, a model was derived for the evolution of ocean frequency-directional spectra for waves propagating shoreward in shoaling waters. From the evolution equations for random surface gravity waves in shallow water, a three-wave collision integral was derived where energy redistribution is by collinear resonant interactions. The shoaling and refraction effects were considered through the application of the geometrical optics, or WKB approximation. Nonlinearities and linear shoaling and refraction effects were combined, in the form of a wave Boltzmann equation that constitutes the basic model equation. The evolution of the spectral energy was obtained by integrating numerically the wave Boltzmann equation with a piecewise ray method and a linear discretization formulation for the three-wave collision integral.

To test the wave spectral transformation model (WST) against experimental data, we required high resolution frequency-directional wave spectra. Combined low resolution of frequency and direction in the initial conditions can distort the simulated evolution of the wave spectra by artificially introducing amplitude dispersion effects. The only known data set with adequate resolution for testing the performance of the WST model is that of Freilich, Guza, and Elgar (1990) (hereinafter FGE90). The observations refer to measurements at Torrey Pines Beach, California (Fig. 3a). We digitized the FGE90 published data for use here (Fig. 3b). The resulting synthetic data set has limitations associated with the limited extent of the information available, the errors inherent to a digitizing process, and to numerical interpolation. Within such conditions, any quantitative evaluation of the model performance has questionable formal validity and one must be cautious of the conclusions drawn. However, qualitative evaluation of the potential of the WST model is possible.

The spectrum was propagated for 256 m over a planar bathymetry which reasonable represents the field conditions. The model results for the frequency-directional spectrum at 4 meter depth are shown in Fig. 4. The most obvious nonlinear effects that can be noticed in the observations of FGE90 at 4 meter depth (Fig. 4a) are the peaks of energy centered at .16 Hz, +6 degrees (coupling of .06 with .10 Hz), and at .18 Hz, -4 degrees (coupling of .06 with .12 Hz). Also, an enhancement of the peak of energy centered at .12 Hz, -4 degrees (self-coupling of.06 Hz) can be observed. The comparison of the frequency-directional spectrum predicted by the WST model (Fig 4b) with FGE90 observations at 4 meter depth shows good agreement. For the sector of waves approaching the beach from the southern quadrant (negative directions), both the peak of energy at .18 Hz, the enhancement of energy at .12 Hz, and the general distribution of energy with direction, compare well. For waves from the northern quadrant (positive directions), the peak of energy at .16 Hz is predicted, but in a direction closer to the beach normal.

The frequency spectra of the simulations and the observations at 4 meter depth are compared in Figure 5. The model results are not statistically different from the observations for most of the frequency range (95 percent confidence level for the spectral estimates based on 320

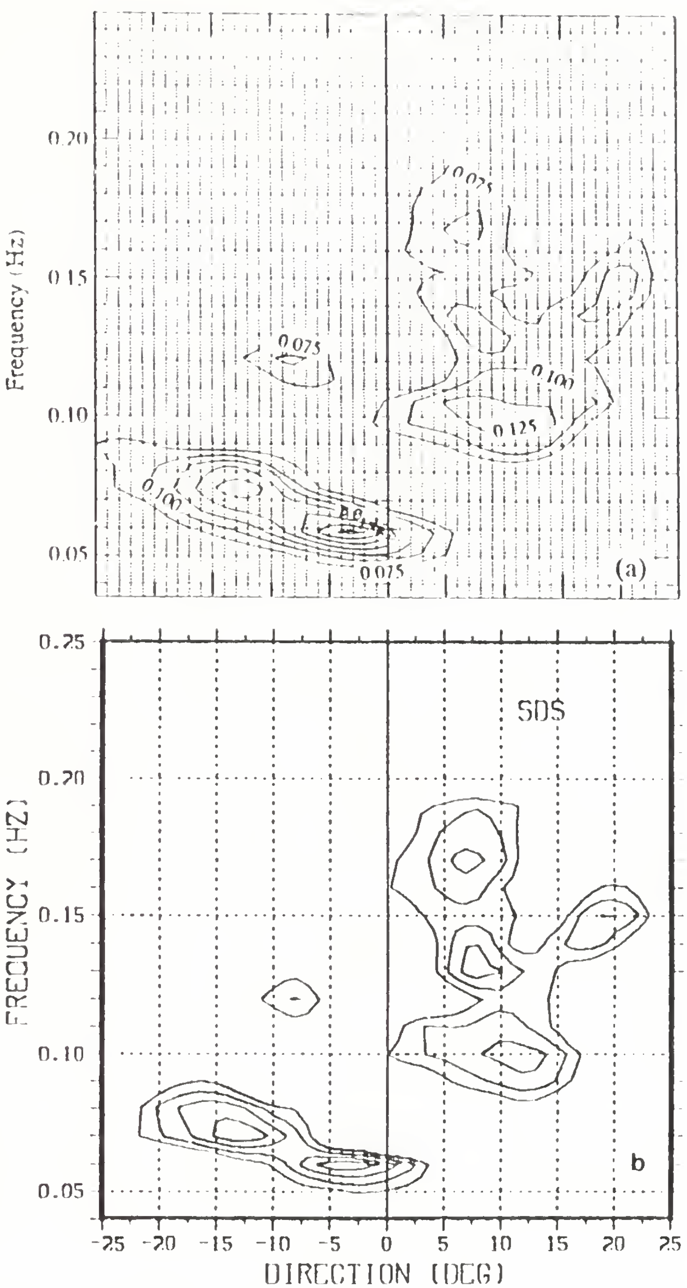

FIGURE 3. The synthetic data set (SDS) for frequency-directional wave spectrum (b) reasonably represents the observations at 10 meter depth (a). At each frequency the area under the curve is proportional to the spectral energy density at that frequency. For the synthetic data set, the exterior contour has a value of .9, with the contours .2 units apart. Directions are relative to the beach normal. [*(a) is adapted from Freilich et al. 1990, copyright American Geophysical Union*]

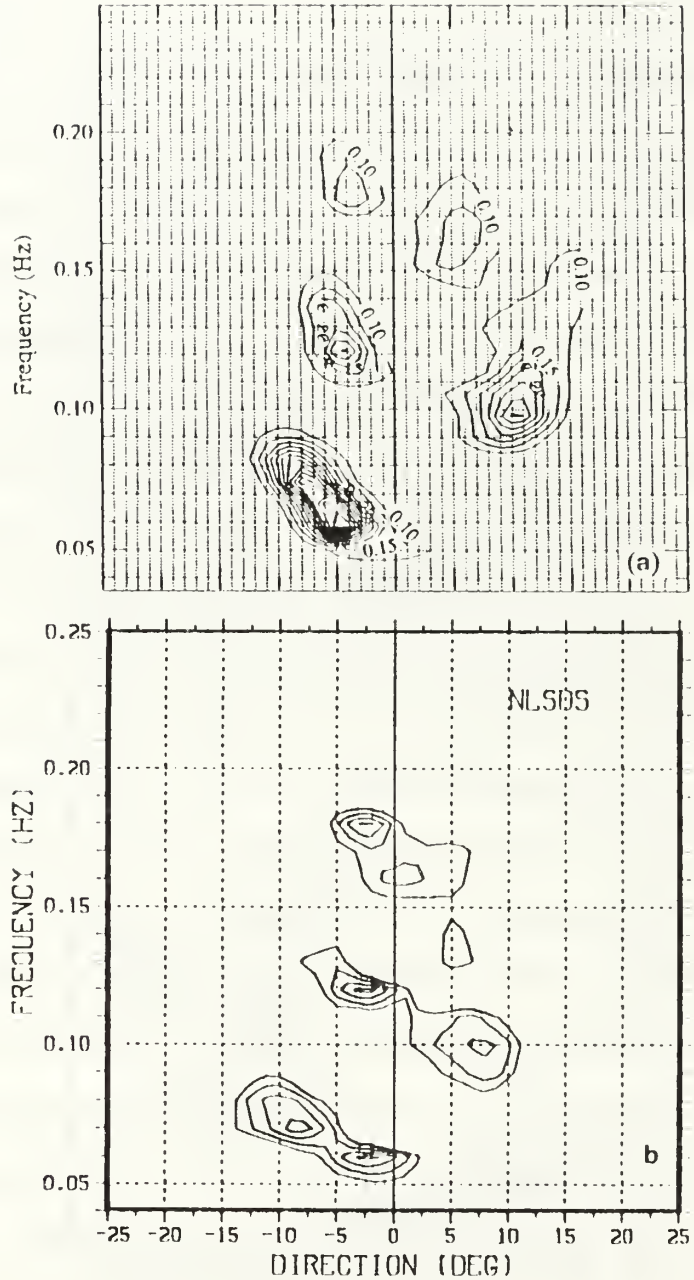

FIGURE 4. The nonlinear evolution of SDS frequency-directional spectrum from 10 to 4 meter depth computed with the WST model. (b) shows that the major features of the observed nonlinear evolution (a) are predicted. The simulated evolution of waves incoming from the north are dominated by nonlinear interactions between harmonic frequencies (.06, .12 Hz), while for waves from the south, the most evident nonlinear effect is the peak of energy centered at .16 Hz. At each frequency (both in (a) and (b)) the area under the curve is proportional to the autospectral density at that frequency. Directions are relative to the beach normal. Exterior contour for (b) has a value of 1.4, and the contour interval is .2. [*(a) is adapted from Freilich et al. 1990, copyright American Geophysical Union*]

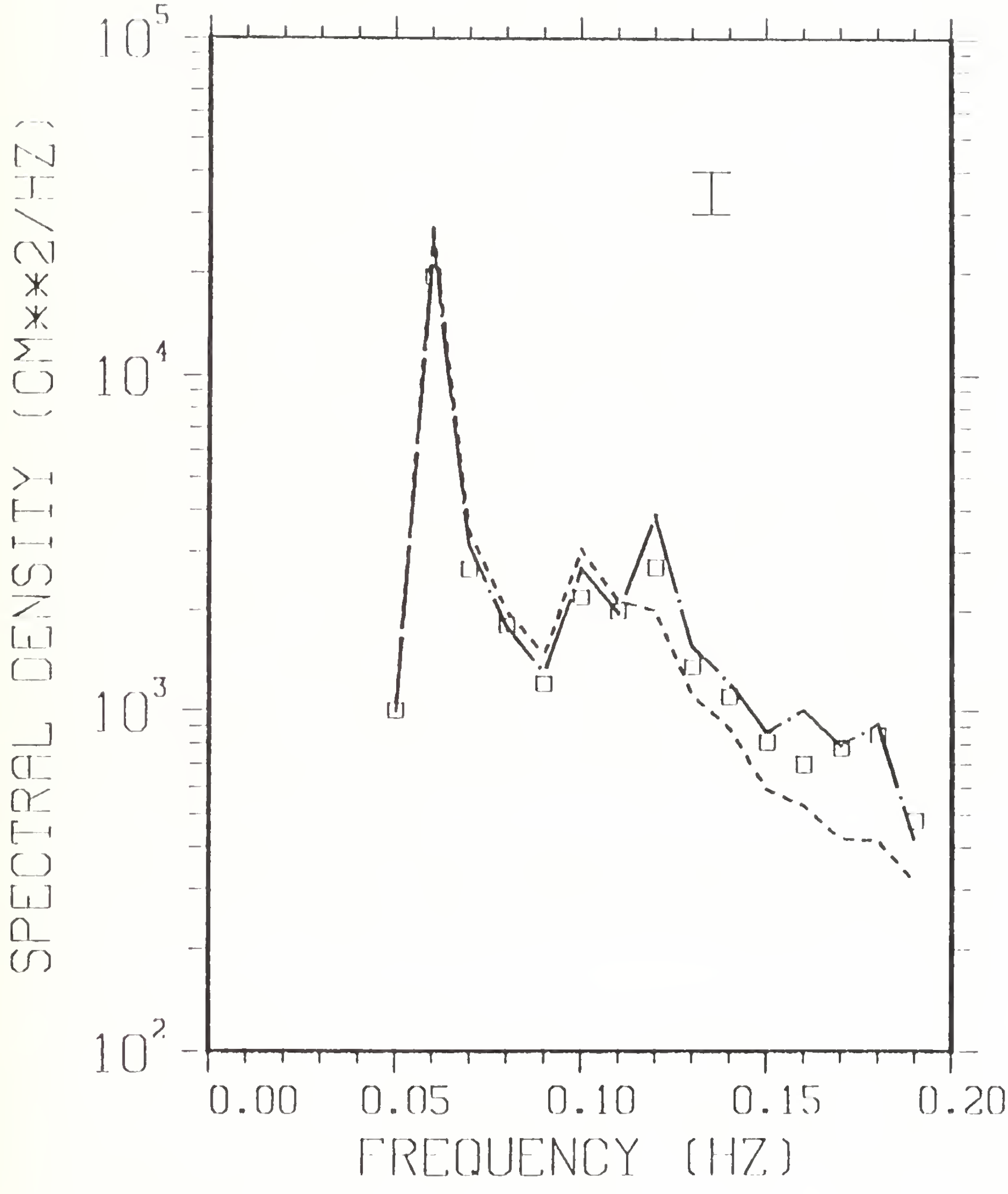

FIGURE 5. WST model results (chain dotted) of frequency spectrum. The model results compare well with the observations (square symbol) in the region of the spectrum dominated by nonlinear effects where linear theory (dashed) is inadequate. The bar represents 95% confidence level for the spectral estimates based on 320 degrees of freedom (Freilich et al. 1990)

degrees of freedom (FGE90)). The simulations show considerable transfer of energy to frequencies .12 Hz and above. This range of frequencies corresponds to the region of the spectrum where linear wave theory is inadequate. The model results are in general superior to estimates using linear, finite depth wave theory, and they compare well with the observations in the region of the spectrum dominated by nonlinear effects. All the prominent peaks of energy due to nonlinear wave interactions (at .12, .16 and .18 Hz) are predicted by the WST model.

By using a geometrical-optics approach we have negated diffraction effects. Such effects become important at wavelengths larger than the characteristic length of variations of the medium. On the other hand, the amplitude ordering adopted for nonlinear interaction of three waves is independent of the ordering appropriate to describe geometrical variations in the bathymetry. One might therefore wonder if diffraction and refraction effects can be incorporated by using a transfer function formulation, thereby replacing the Boltzmann equation description, but still keeping the collision integral. This model would have the obvious advantage of being able to deal with the nonlinear evolution of directional spectra in shallow water for arbitrary changes in the bathymetry. The most apparent difficulty of this approach would be that the integral for a single ray path has to be replaced by an integral over all paths. This is the subject of current research not only because of the fundamental significance of combining diffraction and nonlinear interaction effects, but also for its large potential for applications.

This project was part of the Ph.D. dissertation of Lt. M. Abreu (Portuguese Navy) under the joint supervision of A. Larraza and E. Thornton, with the latter a faculty member of the Oceanography Department. A manuscript, "Nonlinear transformation of directional wave spectra in shallow water," has been submitted (Dec 91) to the *Journal of Geophysical Research.*

3. Wave Turbulence

A small amount of support went to our wave turbulence experiment. We are looking for a new, collective mode of oscillation in the *density* of random wind-generated waves on the surface of water — “a wave of waves”. The physical picture we would like to establish (if true) is that random waves on the ocean can move as a collective unit, much like the collective motion of a gas is the macroscopic average of the motions of its atoms. Our approach is to create a random mix of surface waves, the analog of the atoms, by blowing wind over a long tank; the steady state distribution of waves is then perturbed by a paddle at one end of the tank. The paddle will be driven with a pulse of random oscillations, locally injecting excess wave energy at its end of the tank, analogous to a pulse of pressure. Transducers spaced along the tank will determine whether the excess energy 1) propagates as a collective pulse down the tank exchanging energy among the waves of the background, in the same way that a pulse of sound exchanges energy between the atoms of a gas; 2) diffuses down the tank as the pulse looses its coherence to the background; or 3) propagates as independent linear waves, in analogy to the ballistic propagation of atoms in a Knudsen gas.

During the time the wave turbulence experiment was being supported by this contract, transducers and electronics where developed that can reliably measure low amplitude spectral components, the paddle and its drive were modified to allow for the injection of random waveforms, and software was written for data collection. One student, Mr. Kerry Yarber, is finishing his Masters degree, and another, LT Richard Lawrence, is being brought on line.

REFERENCES

Freilich, M.H., Guza, R.T., and Elgar, G.L., 1990, "Observations of nonlinear effects in directional spectra of shoaling gravity waves," *J. Geophys. Res.* **95**, 9645-9656.

Izumiya, T., and Horikawa, K., 1987, "On the transformation of directional waves under combined refraction and direction," *Coast. Eng. Japan*, **30**, 49-65.

Liu, P.L.F., Yoon, S.B., and Kirby, J.T., 1985, "Nonlinear refraction-diffraction in shallow water," *J. Fluid Mech.*, **153**, 185-209.

Longuet-Higgins, M.S., 1957, "On the transformation of a continuous spectrum by refraction," *Proc. Cambridge Philos. Soc.*, **53**, 226- 229.

Mollenauer L. F., 1986, "The future of fiber communications: Solitons in an all-optical system," Optics News, May 1986, 42-46.

Manuscripts

Paper submitted for publication in refereed journals:

M. Abreu, A. Larraza, and E. Thornton, "Nonlinear Transformation of Directional Wave Spectra in Shallow Water," submitted to *Journal of Geophysical Research*. Other support: NPS direct (internal) funds, and ONR Coastal Science Program.

Paper published in refereed journal:

Bruce Denardo, Brian Galvin, Alan Greenfield, Andres Larraza, Seth Putterman, and William Wright, "Observations of Localized Structures in Nonlinear Vibratory Lattices: Domain Walls and Kinks," *Physical Review Letters*, **68**, 1730-1733 (1992). Other support: ONT/ASEE Postdoctoral Fellowship.

Technical Reports (student theses):

Manuel Abreu, "Nonlinear Transformation of Directional Wave Spectra in Shallow Water," (Ph.D thesis). DTIC number not yet available. Other support: NPS direct (internal) funds, and ONR Coastal Science Program.

Brian Galvin, "Numerical Studies of Localized Vibrating Structures in Nonlinear Lattices," (M.S. thesis). DTIC number not yet available. Other support: ONT/ASEE Postdoctoral Fellowship.

Steven Alkov, "Multifrequency Acoustic Resonators with Variable Nonuniformity," (M.S. thesis). DTIC number not yet available. Other support: ONT/ASEE Postdoctoral Fellowship.

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